Technical Comments

Comments on "Transformations for Infinite Regions and Their Applications to Flow Problems"

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THE purpose of this Comment is to further the discussion of mapping functions used to transform infinite regions into finite intervals initiated by Sills in Ref. 1. In particular, the comparison of the algebraic transformations used in this article with the velocity-related transformations used in Refs. 2 and 3 will be extended. All of these transformations provide a coordinate essentially normal to the flow direction that remains finite even when the actual physical coordinate approaches infinity. When this is achieved, highly successful finite-difference techniques implicit in the normal coordinate can be employed to solve various viscous flow problems numerically. The velocity-related transformations were derived from some bounded property of the momentum equation. For example, the Crocco transformation to velocity as the new independent variable is useful for studying bounded monotonic velocity profiles.[2] Similarly, transformation to a momentum coordinate is useful for studying jet flows.[3]

A particular algebraic transformation can be applied to a greater number of different fluid dynamic problems than a particular velocity-related transformation because it is independent of the velocity characteristics of any particular flowfield. For example, the Crocco transformation is useful for studying bounded monotonic velocity profiles only, whereas the analogous algebraic transformation of Ref. 1 is applicable regardless of the nature of the resulting velocity profile. However, a disadvantage which results from this independence is that as the solution proceeds downstream and the viscous layer grows laterally, it tends to "outgrow" the finite-difference grid of the algebraic transformation, which remains essentially fixed in space. That is, increasingly more of the flowfield is contained within the grid point corresponding to an infinite physical coordinate and the grid point immediately preceding that one. For this reason, the grid must be periodically revised as the solution continues downstream in order to properly describe the flowfield. If, however, the transformed independent variable is some bounded property of the flow, then the finite-difference grid grows "automatically," with the flowfield, and artificial downstream revisions are unnecessary. The numerical solutions of Refs. 2 and 3 enjoyed this advantage.

The velocity-related transformations lose effectiveness, however, if solutions of the energy or species equations are required in addition to the momentum equation, and the Prandtl or Schmidt numbers, respectively, are less than unity. For these cases, the enthalpy or concentration profiles extend further into the infinite region than the velocity profile. As a result, the derivatives of these quantities with respect to the velocity-related independent variable are unbounded at the transformed infinite boundary, and the velocity-related transformation fails there. Flowfield solutions using the algebraic transformations of Ref. 1 do not experience this difficulty.

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References

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Comment on "Effects of a Dynamic Gas on Breakdown Potential"

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IN Ref. 1, the author describes an experiment in which a pair of metal electrodes were attached to the downstream end of a converging, plastic nozzle. Nonionized argon flowed transverse to the electric field at the electrodes. The argon discharged as a freejet into a vacuum chamber at the downstream end of the electrode pair. The author reports that, for fixed gas pressure in the interelectrode region, the presence of a gas flow produces a sharp reduction in the breakdown voltage of the argon below the static value. He concludes that the convective effect of a transverse gas flow *reduces* the gas breakdown potential. This conclusion is generally incorrect. The critical electric field E^* for breakdown of a nonionized gas in the presence of a transverse gas flow of velocity V is given by Eq. (10) of Ref. 2 as follows:

$$\alpha\mu_e E^* = V^2/4D_e + V_e^2/4D_e \tag{1}$$

where α is the first Townsend ionization coefficient; and μ_e D_e and V_e are the electron mobility, diffusion coefficient, and drift velocity, respectively. An additional term involving D_e which appears on the right-hand side of Eq. (1) has been neglected because it is generally small.[2] Equation (1) shows that a transverse gas flow *increases* the breakdown electric field. For the experimental conditions of Ref. 1, $V \approx 10^4$ cm/sec and $V_e > 10^6$ cm/sec.[3] Thus, the gas flow should have a negligible effect on the breakdown process. A probable explanation for the observed reduction in breakdown

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